Propagation of ultrahigh energy cosmic rays and compact sources

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Abstract

The clustering of ultrahigh energy ($> 10^{20}$ eV) cosmic rays (UHECR) suggests that they might be emitted by compact sources. Statistical analysis (Dubovsky et al., 2000) estimated the source density. We extend their analysis to give also the confidence intervals (CI) for the source density using a.) no assumptions on the relationship between clustered and unclustered events; b.) nontrivial distributions for the source luminosities and energies; c.) the energy dependence of the propagation. We also determine the probability that a proton created at a distance r with energy E arrives at earth above a threshold E_c. Using this function one can determine the observed spectrum just by one numerical integration for any injection spectrum. The observed 14 UHECR events above 10^{20} eV with one doublet gives for the source densities $180^{+2730}_{-165} \cdot 10^{-3}$ Mpc^{-3} (on the 68% confidence level).

1 INTRODUCTION

The interaction of protons with photons of the cosmic microwave background predicts a sharp drop in the cosmic ray flux above the GZK cutoff around $5 \cdot 10^{19}$ eV (Greisen, 1966; Zatsepin and Kuzmin, 1966). The available data shows no such drop. About 20 events above 10^{20} eV were observed by a number of experiments such as AGASA (Takeda et al., 1998), Fly's Eye (Bird et al., 1993), Haverah Park (Lawrence et al., 1991), Yakutsk (Efimov et al., 1991) and HiRes (Kieda et al., 1999). Future experiments, particularly Pierre Auger (Boratav, 1996; Guerard, 1999; Bertou et al. 2000), will have a much higher statistics. Since above the GZK energy the attenuation length of particles is a few tens of megaparsecs (Yoshida and Teshima, 1993 ; Aharonian and Cronin, 1994; Protheroe and Johnson, 1996; Bhattacharjee and Sigl, 2000; Achterberg et al., 1999; T. Stanev et al., 2000) if an ultrahigh energy cosmic ray (UHECR) is observed on earth it is usually assumed that it is produced in our vicinity.

At these high energies the galactic and extragalactic magnetic fields do not affect the orbit of the cosmic rays, thus they should point back to their origin within a few degrees. In contrast to the low energy cosmic rays one can use UHECRs for point-source search astronomy. Though there are some peculiar clustered events, which we discuss in detail, the overall distribution of UHECRs on the sky is practically isotropic. This observation is rather surprising since in principle only a few astrophysical sites are capable to accelerate such particles.

There are several ways to look for the source inhomogeneity from the energy spectrum and spatial directions of UHECRs. One possibility is to assume that the source density of UHECRs is proportional to the galaxy densities (Waxman et al., 1997; Giller et al., 1980; Hill and Schramm, 1985); or one can analyze the clustering of the unknown sources by some correlation length (Bahcall and Waxman 2000).

Clearly, the arrival directions of the UHECRs measured by experiments show some peculiar clustering: some events are grouped within $\sim 3^o$, the typical angular resolution of an experiment. Above $4 \cdot 10^{19}$ eV 92 cosmic ray events were detected, including 7 doublets and 2 triplets. Above 10^{20} eV one doublet out of 14 events were found (Uchihori et al., 2000). The chance probability of such a clustering from uniform distribution is rather small (Uchihori et al., 2000; Hayashida et al., 1996; Tinyakov and Tkachev 2001a, Tinyakov and Tkachev 2001b,).

The clustered features of the events initiated an interesting statistical analysis assuming compact UHECR sources (Dubovsky et al., 2000). The authors found a large number, ~ 400 for the number of sources inside a GZK sphere of 25 Mpc. We extend their analysis to give also the CIs for the source density using a.) no assumptions on the relationship between clustered and unclustered events; b.) nontrivial distributions for the source luminosities and energies; c.) the energy dependence of the propagation. We also determine the probability that a proton created at a distance r with energy E arrives at earth above a threshold E_c.

As we show the most probable value for the source density is really large; however, the statistical significance of this result is rather weak. At present the small number of UHECR events allows a 95% CI for the source density which spreads over four orders of magnitude. Since future experiments, particularly Pierre Auger, will have a much higher significance on clustering (the expected number of events of 10^{20} eV and above is 60 per year, we present give results also for larger number of UHECRs above 10^{20} eV.

In order to avoid the assumptions of (Dubovsky et al., 2000) a combined analytical and Monte-Carlo technique will be presented adopting the conventional picture of protons as

the ultrahigh energy cosmic rays. Our analytical approach of Section 2 gives the event clustering probabilities for any space, luminosity and energy distribution by using a single additional function $P(r, E; E_c)$, the probability that a proton created at a distance r with energy E arrives at earth above the threshold energy E_c (Bahcall and Waxman, 2000). With our Monte-Carlo technique of Section 3 we determine the probability function $P(r, E; E_c)$ for a wide range of parameters. Our results for the present and future UHECR statistics are presented in Section 4. We summarize in Section 5.

2 ANALYTICAL APPROACH

The number of UHECRs emitted by a source of λ luminosity during a period T follows the Poisson distribution. However, not all emitted UHECRs will be detected. They might loose their energy during propagation or can simply go to the wrong direction. For UHECRs the energy loss is dominated by the pion production in interaction with the cosmic microwave background radiation. In a recent analysis (Bahcall and Waxman, 2000) the probability function $P(r, E, E_c)$ was presented for three specific threshold energies. This function gives the probability that a proton created at a given distance from earth (r) with some energy (E) is detected at earth above some energy threshold (E_c).

The features of the Poisson distribution enforce us to take into account the fact that the sky is not isotropically observed.

The probability of detecting k events from a source at distance r with energy E can be obtained by simply including the factor $P(r, E, E_c)A\eta/(4\pi r^2)$ in the Poisson distribution:

$$p_k(\mathbf{x}, E, j) = \frac{\exp\left[-P(r, E, E_c)\eta j/r^2\right]}{k!} \times \left[P(r, E, E_c)\eta j/r^2\right]^k, \qquad (1)$$

where we introduced $j = \lambda T A/(4\pi)$ and $A\eta/(4\pi r^2)$ is the probability that an emitted UHECR points to a detector of area A. The factor η represents the visibility of the source, which was determined by spherical astronomy. We denote the space, energy and luminosity distributions of the sources by $\rho(\mathbf{x})$, $c(E)$ and $h(j)$, respectively. The probability of detecting k events above the threshold E_c from a single source randomly positioned within a sphere of radius R is

$$P_k = \int_{S_R} dV\ \rho(\mathbf{x}) \int_{E_c}^{\infty} dE\ c(E) \int_0^{\infty} dj\ h(j) \times \frac{\exp\left[-P(r, E, E_c)\eta j/r^2\right]}{k!} \left[P(r, E, E_c)\eta j/r^2\right]^k. \qquad (2)$$

Denote the total number of sources within the sphere of sufficiently large radius (e.g. several times the GZK radius) by N and the number of sources that gave k detected events by N_k. Clearly, $N = \sum_0^\infty N_i$ and the total number of detected events is $N_e = \sum_0^\infty iN_i$. The probability that for N sources the number of different detected multiplets are N_k is:

$$P(N, \{N_k\}) = N! \prod_{k=0}^{\infty} \frac{1}{N_k!} P_k^{N_k}. \qquad (3)$$

For a given set of unclustered and clustered events (N_1 and N_2, N_3,...) inverting the $P(N, \{N_k\})$ distribution function gives the most probable value for the number of sources and also the CI for it.

Note, that P_k and then $P(N, \{N_k\})$ are easily determined by a well behaving four-dimensional numerical integration for any $c(E)$, $h(j)$ and $\rho(r)$ distribution functions. In order to illustrate the uncertainties and sensitivities of the results we used a few different choices for these distribution functions.

For $c(E)$ we studied three possibilities. The most straightforward choice is the extrapolation of the 'conventional high energy component' $\propto E^{-2}$. Another possibility is to use a stronger fall-off of the spectrum at energies just below the GZK cutoff, e.g. $\propto E^{-3}$. The third possibility is to assume that UHECRs are some decay products of metastable superheavy particles (Berezinsky et al., 1997; Kuzmin and Rubakov, 1998; Birkel and Sarkar, 1998; Sarkar 2000; Fodor and Katz 2001b). According to (Birkel and Sarkar, 1998) the superheavy particles decay into quarks and gluons which initiate multi-hadron cascades through gluon bremstrahlung.

In the recent analysis (Dubovsky et al., 2000) the authors have shown that for a fixed set of multiplets the minimal density of sources can be obtained by assuming a delta-function distribution for $h(j)$. We studied both this limiting luminosity: $h(j) = \delta(j - j_*)$ and a more realistic one with Schechter's luminosity function (Schechter 1976), which can be given as: $h(j)dj = h \cdot (j/j_*)^{-1.25} \exp(-j/j_*) d(j/j_*)$.

The space distribution of sources can be given based on some particular survey of the distribution of nearby galaxies or on a correlation length r_0 characterizing the clustering features of sources. For simplicity the present analysis deals with a homogeneous distribution of sources.

3 MONTE-CARLO STUDY OF THE PROPAGATION

In our Monte-Carlo approach we determined the propagation of UHECR protons on an event by event basis. The inelasticity of Bethe-Heitler pair production is small ($\approx 10^{-3}$), thus we used a continuous energy loss approximation for this process. The inelasticity of pion-photoproduction is larger ($\approx 0.2 - 0.5$) in the energy range of interest, thus there are only a few tens of such interactions during the propagation. Due to the Poisson statistics and the spread of the inelasticity, we will see a spread in the energy spectrum even if the injected spectrum is mono-energetic.

In our simulation protons are propagated in small steps (10 kpc), and after each step the energy losses due to pair production, pion production and the adiabatic expansion are calculated. During the simulation we keep track of the current

energy of the proton and its total displacement. For the proton interaction lengths and inelasticities we used the values of (Bhattacharjee and Sigl, 2000; Achterberg et al., 1999). The deflection due to magnetic field is not taken into account (for a recent Monte-Carlo on it see eg. Stanev et al., 2000).

To cover a broad energy range we used the parametrization

$$P(r, E, E_c) = \exp\left[-a \cdot (r/1\,\mathrm{Mpc})^b\right]. \qquad (4)$$

Fig. 1 shows the functions $a(E/E_c)$ and $b(E/E_c)$ for a range of three orders of magnitude and for five different threshold energies. Just using the functions of $a(E/E_c)$ and $b(E/E_c)$, thus a parametrization of $P(r, E, E_c)$ one can obtain the observed energy spectrum for any injection spectrum without additional Monte-Carlo simulation.

4 RESULTS

In order to determine the CIs for the source densities we used the frequentist method (Groom et al., 2000). We wish to set limits on S, the source density. Using our Monte-Carlo based $P(r, E, E_c)$ functions and our analytical technique we determined $p(N_1, N_2, N_3, ...; S; j_*)$, which gives the probability of observing N_1 singlet, N_2 doublet, N_3 triplet etc. events if the true value of the density is S and the central value of luminosity is j_*. For a given set of $\{N_i, i = 1, 2, ...\}$ the above probability distribution as a function of S and j_* determines the 68% and 95% confidence level regions in the $S-j_*$ plane. For different choices of $c(E)$ and $h(j)$ see Table 1 for the calculated densities. Our results for the Dirac-delta luminosity distribution are in agreement with the result of (Dubovsky et al., 2000) within the error bars. Neverthless, there is a very important message. The CIs are are so large that on the 95% confidence level two orders of magnitude smaller densities than suggested as a lower bound by (Dubovsky et al., 2000) are also possible.

It is of particular interest to study higher statistics too, and determine CIs for these cases. We performed a detailed analysis on this question (Fodor and Katz 2001a). An interesting feature of the results is that "doubling" the present statistics with the same clustering features (in the case studied by the table this means one new doublet out of 10 new events) reduces the CIs by an order of magnitude. The study of even higher statistics leads to the conclusion that experiments in the near future with approximately 200 UHECR events can tell at least the order of magnitude of the source density.

5 SUMMARY

We presented a technique in order to statistically analyze the clustering features of UHECR. The technique can be applied for any model of UHECR assuming small deflection. The key role of the analysis is played by the P_k functions defined by eqn. (2), which is the probability of detecting k events above the threshold from a single source. Using a combinatorial expression of eqn. (3) the probability distribution for

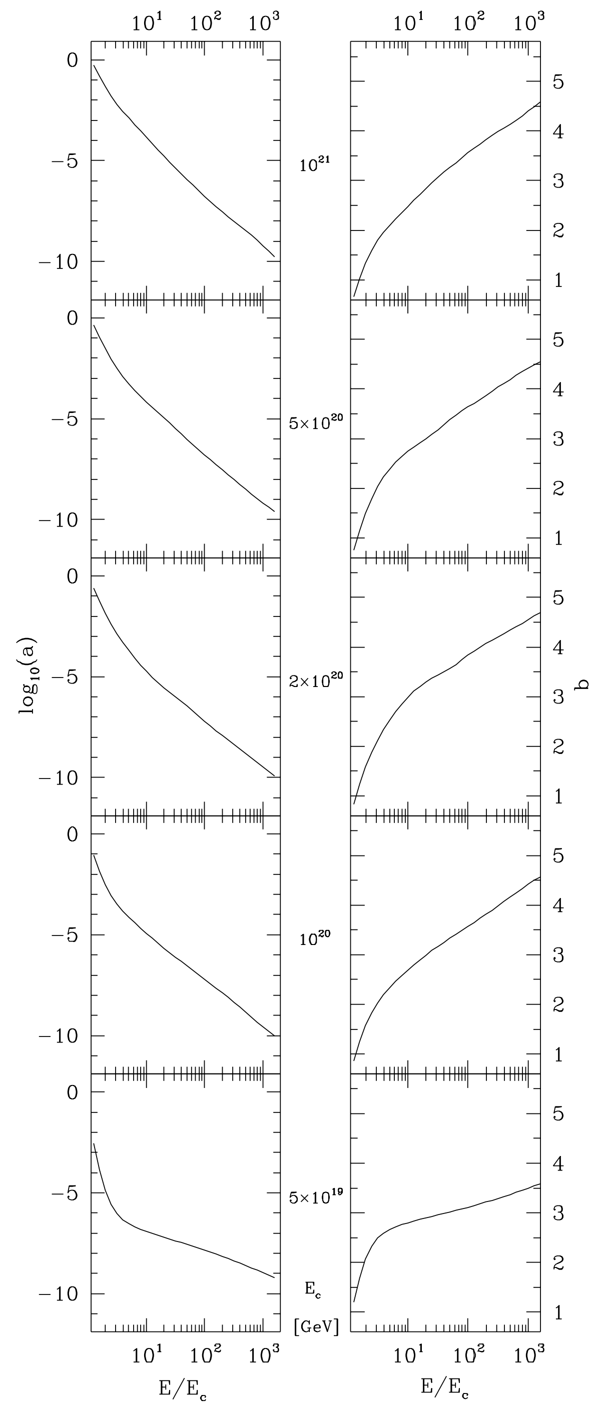

Figure 1: *The functions* $a(E/E_c)$ *–left panel– and* $b(E/E_c)$ *–right panel– for the probability distribution function* $P(r, E, E_c)$ *using the parametrization* $\exp[-a \cdot (r/1\,\mathrm{Mpc})^b]$ *for five different threshold energies (*$5 \cdot 10^{19}$ *eV,* 10^{20} *eV,* $2 \cdot 10^{20}$ *eV,* $5 \cdot 10^{20}$ *eV and* 10^{21} *eV).*

$c(E)$	$h(j)$	**14 events 1 doublet**
$\propto E^{-2}$	$\propto \delta$	$2.77^{+96.1(916)}_{-2.53(2.70)}$
$\propto E^{-2}$	$\propto$ SLF	$36.6^{+844(4268)}_{-34.3(35.9)}$
$\propto E^{-3}$	$\propto \delta$	$5.37^{+80.2(624)}_{-4.98(5.25)}$
$\propto E^{-3}$	$\propto$ SLF	$180^{+2730(8817)}_{-165(174)}$
$\propto$ decay	$\propto \delta$	$3.61^{+116(1060)}_{-3.30(3.51)}$
$\propto$ decay	$\propto$ SLF	$40.9^{+856(4345)}_{-38.3(40.1)}$

Table 1: *The most probable values for the source densities and their error bars given by the 68% and 95% confidence level regions (the latter in parenthesis). The numbers are in units of 10^{-3} Mpc^{-3} The three possible energy spectrums are given by a distribution proportional to E^{-2}, E^{-3}, or by the decay of a 10^{12} GeV particle (denoted by "decay"). The luminosity distribution can be proportional to a Dirac-delta or to Schechter's luminosity function (denoted by "SLF").*

any set of multiplets can be given as a function of the source density.

We discussed several types of energy and luminosity distributions for the sources and gave the most probable source densities with their CIs for present and future experiments.

The probability $P(r, E, E_c)$ that a proton created at a distance r with energy E arrives above the threshold E_c (Bahcall and Waxman, 2000) is determined and parametrized for a wide range of threshold energies. This result can be used to obtain the observed energy spectrum of the UHECR for arbitrary injection spectrum.

In ref. (Dubovsky et al., 2000) the authors analyzed the statistical features of clustering of UHECR, which provided constraints on astrophysical models of UHECR if the number of clusters is small, by giving a bound from below. In our paper we have shown that there is some constraint, but it is far from being tight. At present statistics the 95% CIs usually span 4 orders of magnitude. Two orders of magnitude smaller numbers than the prediction of (Dubovsky et al., 2000) (their eqn. (13) suggests for the density of sources $\sim 6 \cdot 10^{-3}$ Mpc^{-3}) can also be obtained. Adding 10 new events with an additional doublet the CI can be reduced to 3 orders of magnitude and the increase of the UHECR events to 200 can tell at least the order of magnitude of the source density.

More details of the present analysis can be found in (Fodor and Katz, 2001a). Note, that a similar study based on the Z-burst scenario (Fargion et al., 1999; Weiler, 1999) can be carried out which gives the mass of the heaviest neutrino (Fodor et al., 2001).

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